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TRANSLATIONS OF ABSTRACTS OF SHORT COMMUNICATIONS FROM THE RUSSIAN ORIGINALS

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Only a few of the following translations have been given by the authors. Many errors may occur in the translation of the others, for the text is very condensed and therefore difficult to interpret. However, it is hoped that the translation may still serve a useful purpose.

Adjan, S. I., The cancellation law in finitely defined semi-groups.

The role of a cancellation law when giving semi-groups by means of defining relations is investigated. An example is constructed of a semi-group which is given with cancellation law (laws) and a finite number of defining relations and which cannot, without a cancellation law, be given by a finite number of defining relations. It is also proved that every semi-group with cancellation laws and a finite number of defining relations can be effectively imbedded in a semi-group which can be given by a finite number of defining relations without the use of a cancellation law.

Aleksandrjan, R. A., On some systems of S. L. Sobolev's type and on homogeneous boundary problems for differential operators with indefinite quadratic form.

Consider a homogeneous boundary problem for differential operators of the type

$$\text{div}(A + \lambda E)B \text{ grad } U = 0,$$

whose quadratic form is indefinite.

It is shown that this last problem can be studied by means of a perturbation of the left-hand side by the differential expression $\pm i\tau \text{ div } B(x) \text{ grad } U$ followed by a passage to the limit.

In the most typical particular case, which corresponds to the homogeneous Dirichlet problem for the equation of a vibrating string

$$(1 + \lambda)\frac{\partial^2 u}{\partial x^2} - (1 - \lambda)\frac{\partial^2 u}{\partial y^2} = 0, \quad |\lambda| < 1$$

a family of special automorphisms S_λ of the boundary Γ of the given domain is considered. Let $A(\lambda_o, \Gamma)$ be the set of points on Γ, which are fixed with respect to some iteration of the automorphism S_λ. It turns out that if $A(\lambda_o, \Gamma)$ is non-empty, then there exist non-trivial generalized solutions of the homogeneous problem. If the set $A(\lambda_o, \Gamma)$ has interior points, then there exist arbitrarily many linearly independent principal solutions. Similar conditions are sufficient to make the spectrum of the corresponding operator continuous in some interval.

It is proved that if the automorphism S_{λ_o} is ergodic (the set $A(\lambda_o, \Gamma)$ empty), then in the class of measurable functions there does not exist any non-trivial solution of the homogeneous problem. If, however, the automorphism S_{λ_o} is not ergodic, then there is a method to construct all the generalized solutions of the equation of the string, vanishing at the boundary of the domain, in the class of piecewise continuous functions.

<u>Andrunakievič, V.A., Radicals and the decomposition of a ring</u>.

Let R be an overnilpotent radical of a ring, and R_c a radical complementary to R. [1] We will say that the ring K is bounded by the radical R, if $R^* \subseteq R$, where $R^* = \{a \in K | aR = Ra = 0\}$. Rings with a minimal condition for principal right ideals will be called MP rings.

<u>Theorem</u>. Let R = R(K) be a given overnilpotent radical of the ring K. If $\overline{K} = K/R$ is a MP ring, then $K = R_c \dotplus R_c^*$, where either $R_c = 0$, or R_c is a discrete direct sum R of semi-simple MP rings. In addition, $R(R_c^*) = R(K)$, and the ring R_c^* is bounded by the radical R. Finally, either $R_c^* = R$, or R_c^*/R is a discrete direct sum R of semi-simple prime MP rings.

This generalizes Veit's theorem which is obtained in case R = I, where I is Jacobson's radical.

<u>Corollary</u>. Every subdirectly indecomposable MP ring with nilpotent kernel is bounded by Baire's radical.

[1] Literature: V.A. Andrunakievič, Matem. Sb., 44(86):2, (1958), 179.

<u>Anosov, D.V., The roughness of geodesic currents in compact Riemannian manifolds of negative curvature</u>.

Let V be a compact Riemannian manifold of negative curvature (the curvature is supposed to be negative at every point and in an arbitrary two-dimen-

sional direction), and let W be the space of all its linear elements. The geodesic lines of V define a dynamic system ("geodesic current") in W. It turns out that this dynamic system is rough (in the sense of Andronov and Pontrjagin).

Arhangelskiǐ, A.K., On the dimension of spaces.

The rank $r_x B$ of a system of sets B is $< k$ at the point x if among k arbitrary elements of the system B, which contain x, there is one contained in another.

Theorem. A normal space X has dimension $\dim X < k$ if and only if in an arbitrary covering of X it is possible to inscribe a covering η such that $r_x \eta < < k + 1$, $x \in X$.

If in defining the rank we confine ourselves to sets in B, contained in some $O_x \ni x$, we get $r_x^{O_x} B$.

Put $locr_x B = \min_{O_x \ni x} r_x^{O_x} B.$

Theorem. For a metric space X to have $\dim X < k$ it is necessary and sufficient that there exists a basis B in X, for which $locr_x B < k + 1$, $x \in X$.

Theorem. For a normal X to be weakly countably dimensional, it is necessary and sufficient that there exists on X a function $\varphi(x)$ such that in an arbitrary (finite) covering of X, it is possible to inscribe a covering η such that $\varphi_\eta(x) < < \varphi(x)$, $x \in X$, where $\varphi_\eta(x)$ is the multiplicity function of the covering η.

Arnol'd, V.I., Perturbation theory and the problem of stability for planetary systems.

The recent progress of perturbation theory [1 - 5] permits us, in particular, to establish that if the masses of the planets are sufficiently small in comparison with the mass of the central body, then for the majority of initial conditions, under which the eccentricities and inclinations of the orbits are sufficiently small, the motion is conditionally periodic, and the magnitudes of the major semi-axes remain near their initial values.

Literature:
1. A.N. Kolmogorov, DAN 98 No 4 (1954), 527-530.
2. V.I. Arnol'd, DAN 137 No 2 (1961), 255-257.
3. V.I. Arnol'd, DAN 138 No 1 (1961), 13-15.
4. V.I. Arnol'd, DAN 142 No 4 (1961), 758-761.
5. V.I. Arnol'd, DAN   (1962).

Bicadze, A.V., On the equations of mixed type in three independent variables.

The purpose of this report is to state and solve some boundary value problems for the equations of mixed elliptic-hyperbolic type in three independent variables. These problems generalize the well known Tricomi problem.

As model equations

$$\text{sgn} z \cdot u_{xx} + u_{yy} + u_{zz} = g(x, y, z)$$

and

$$z \cdot u_{xx} + u_{yy} + u_{zz} = g(x, y, z)$$

are taken.

Bol'šev, L.N., Some applications of Pearson transformations.

A function of a stochastic variable is called a Pearson transformation of the stochastic variable if its distribution belongs to the K. Pearson family. The theory of Pearson transformation is a consequence of certain limit theorems and asymptotic formulas analogous to the expansions of Edgeworth and Cornish-Fisher (this theory should be regarded as an extension of the theory of normal transformations, developped in the works of R.A. Fisher and V. Vazov). On the basis of studies of the properties of the Pearson transformations, the author has succeeded in establishing some convenient approximations of distributions close to Pearson distributions. The results are useful for the study of asymptotic properties of the beta distribution, and permits the author to indicate a new approximation of the binomial distribution, more exact than the normal or Poisson approximations.

The theory of Pearson transformations can be applied when constructing stochastic variables with given distributions, similar to the distributions of the Pearson family. This field of application is connected with the task of making large sequences of random numbers (in the first place for realization of computations according to the Monte Carlo method).

Čogošvili, G.S., Diagrams of groups with outer multiplication and some of their applications in homology theory.

1) Consider diagrams of graded groups which are connected not only by homomorphisms in the usual way but also by outer products; the degree of homomorphisms (outer products) is understood in some generalized sense. For a directed system of diagrams which satisfy certain conditions of compatiablity, the concept of limit diagram is introduced.

2) A method of construction of homology groups of general spaces is exhibited which is founded on considerations of some special classes of complexes and which gives new types of homology groups beside the known ones.

3) By means of directed systems of diagrams and their limit groups relations are established for projective and spectral groups for arbitrary subsets of a

manifold (which contain e.g. duality relations) and the connection between these relations.

Daniljuk, I.I., On the oblique derivative problem.

The oblique derivative boundary problem for an elliptic system of the first order in the plane is investigated. The method consists in reducing the problem to another one whose boundary conditions do not contain derivatives of the unknown functions.

A complete analysis of the problem in the linear case for an arbitrary multiply connected region is given, the spectrum of the problem is studied, and a few theorems on existence of solutions in the non-linear case are established.

Džrbašjan, M.M., Investigation of some incomplete systems in a complex region.

Let an arbitrary incomplete system of functions be given. The following problems arise:

1. Exhibit a complete interior property of the class of functions admitting an approximation by the given incomplete system in the corresponding region.

2. In a defined sense, complete the given incomplete system.

The author has obtained a complete solution of these two problems for Müntz systems, systems of rational functions on the unit circle orthogonal with respect to a weight function, and for a base system of rational functions.

Efimov, N.V., Surfaces of negative Gaussian curvature.

We consider two-dimensional surfaces of negative Gaussian curvature in three-dimensional Euclidean space.

We establish estimating relations between some quantities in the interior and exterior geometries of the surface. We establish propositions expressing the correctness of the well-known Hilbert theorem on surfaces of constant negative curvature or generalizing this theorem to the case of variable curvature.

Efimov, N.V. and Stečkin, S.B., Čebyšev sets in Banach spaces.

Let X be a Banach space. A set $M \subset X$ is called a Čebyšev set if for each point $x \in X$ there exists in the set M a unique point $y = y(x)$ nearest to x.

Questions about existence of Čebyšev subspaces in Banach spaces are studied. There exists a Banach space that has no Čebyšev subspace (Garkavi).

The properties of arbitrary Čebyšev sets in smooth Banach spaces are studied.

Every bounded compact Čebyšev set is convex (Vlasov). Every approximatively compact Čebyšev set in a locally uniformly convex space is convex.

Faddeev, L.D., Construction of the resolvent of the energy operator for a three particle system and the scattering problem.

The author studies the energy operator of a quantum-mechanical system of three non-relativistic particles with pairwise interaction, which vanishes for great distances. For the resolvent kernel of this operator a representation is obtained that can be used in the whole complex plane of the spectral variable. With help of this representation the absolutely continuous spectrum of the energy operator is studied and a theorem on the expansion of an arbitrary function in terms of the eigenfunctions is proved. We also obtain an asymptotic estimate of the solution of the corresponding non-stationary Schrödinger equation for great values of the time variable. The latter results are used to justify the non-stationary formulation of the problem in the collision theory for the system under consideration and to construct the corresponding unitary scattering operator both for elastic and inelastic scattering.

Gamkredlidze, R.V., On moving optimal regimes.

1. The maximum principle in the theory of optimal processes gives necessary conditions, which are satisfied by optimal controls and trajectories. According to these conditions an optimal process can be determined if it exists. The corresponding existence theorem is due A.F. Filippov.

2. If the conditions of the existence theorem are violated, an optimal control does not exist in general. However, a minimizing sequences of controls always exists. The corresponding sequence of trajectories converges to some absolutely continuous curve. The motion along this curve is called an optimal moving regime, if this motion is not realized by any admissible control.

3. By combining the maximum principle and the existence theorem it is possible to find minimizing sequences and corresponding optimal moving regimes, if the set of admissible values for the control is compact.

Gluškov, V.M., Problems of completeness and self-organization in the abstract theory of automata.

The problem of completeness is usually posed in the structural theory of automata after fixation of one inner alphabet (most often dyadic). To pose this problem within the frames of the abstract theory of automata it is necessary to use a generalized concept of composition and, first of all, a concept of generalized product of automata. There exist several natural definitions of completeness

(isomorphic, homomorphic, and algorithmic completeness). It is possible to establish necessary and sufficient conditions for the completeness of a system of abstract automata in different senses.

The problem of self-organization is also usually posed in the structural theory of automata. It is possible, however, to pose it also within the frame of the abstract theory of automata. In a natural way one can introduce the concepts of entropy of teaching and of entropy of examination. Using these it is possible to obtain quantitative estimates of the capability of self-organization in finite abstract automata.

Gnedenko, B.V., see p. 3.23.

Ibragimov, I.A., On conditions for weak dependence for stationary Gaussian processes.

In recent papers by several authors, different conditions on random processes have been investigated, which express, roughly speaking, independence between the remote past and the remote future of processes. For stationary Gaussian processes these conditions can be formulated in terms of the spectral function. In the present report we give theorems on the properties of the spectral functions of stationary Gaussian processes, satisfying different conditions of weak dependence. For instance, if $f(\lambda)$ is the spectral density of the stationary Gaussian process $\{x_n\}$, which satisfies a condition of strong mixing (M. Rosenblatt, Proc. Nat. Acad. Sci. U.S.A., 42, 1 (1956)), it is necessary that $f(\lambda) = g(\lambda)|P(\lambda)|^2$, where $P(\lambda)$ is a trigonometric polynomial, and $g(\lambda)$ is such that as $h \to 0$

$$\int_0^h \left[g(\lambda + 1) - g(\lambda - 1)\right] dl = 0 \left(\int_0^h g(\lambda + 1)dl\right) \quad \text{uniformly in } \lambda.$$

Ibragimov, I.I., Some extremal problems for linear operators in the class of entire functions of finite order.

Let $W_\sigma^{(p)}$ $(p \geq 1)$ denote the set of entire functions $\varphi_\sigma(z)$ of order $\leq \sigma$, for which

$$\|\varphi_\sigma\|_p = \left(\int_{-\infty}^{+\infty} |\varphi_\sigma(x)|^p dx\right)^{1/p} < +\infty.$$

Further, let m denote the set of linear operators T defined on the set $W_\sigma^{(p)}$ $(p \geq 1)$ for every σ and satisfying still some defining conditions. Assuming T and S to be different linear operators of the set m, we establish a dependence in the form of inequalities between the norms

$$\|T(f)\|_p \quad \text{and} \quad \|S(f)\|_q, \quad \text{where } 1 \leq p \leq q \leq \infty.$$

In the many-dimensional case (for instance, in the three-dimensional case), we

establish a dependence between different norms of the form

$$\|f\|^{(3)}_{P_1, P_2, P_3} = \left\{ \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} |f(x_1, x_2, x_3)|^{P_1} dx_1 \right)^{P_2/P_1} dx_2 \right]^{P_3/P_2} dx_3 \right\}^{1/P_3}$$

which coincides with the norm of the function $f(z_1, z_2, z_3) \in W_{\sigma_1, \sigma_2, \sigma_3}$ $(p \geq 1)$ when $P_1 = P_2 = P_3$.

Ivanov, V.K., On an incorrectly stated problem.

Let X and Y be linear metric spaces. Let A be a continuous operator from X to Y and M a compact subset of X. The equation $Ax = y$, where $x \in X$ is unknown and $y \in Y$ is the given element, is examined in the case when x does not depend continuously on y.

It is shown that the element $x_o \in M$ minimizing on M the functional $f(x) = \rho (Ax, y)$ (the best possible approximation) depends continuously on y in a generalized sense. If M is convex, A linear with inverse A^{-1}, and Y strictly convex, then x_o is a continuous function of y.

Kirillov, A.A. Unitary representations of nilpotent Lie groups.

Let $\mathcal{O}$ be a nilpotent Lie group, G' the linear space, conjugate to the Lie algebra of $\mathcal{O}$ and ρ' the natural representation of $\mathcal{O}$ in G'.

It is proved that the irreducible unitary representations of the group $\mathcal{O}$ are in one-to-one correspondence to the orbit in G' relatively $\rho'(\mathcal{O})$.

In terms of the orbit we obtain simple and immediate answers to the usual questions in representation theory: the relation between representations of groups and subgroups, the decomposition in irreducible components of a tensor product of two irreducible representations, the calculation of the characters, the Plancherel formula. It is remarkable that some of these results in the same formulation prove to be true not only for nilpotent, but also for certain solvable and even semi-simple Lie groups.

Kondrat'ev, V.A., On the differentiability of the solution of an elliptic equation up to the boundary.

We consider the first boundary value problem for an elliptic equation in a bounded region which is not supposed to be smooth. We establish estimates on the rate of convergence of the solution to the boundary values depending on the capacity of the complement of the region in a neighborhood of a fixed point. From these estimates it is possible to determine what smoothness the solution will have up to the boundary in the presence of corner and conic points. If the boundary is a corner or a cone of angle a, then for infinitely differentiable boundary functions

and right-hand side, the solution will have $\frac{K}{a}$ derivatives up to the boundary, where K is a constant depending only on the coefficients of the principal part of the equation.

Kudrjavcev, L.D., Weighted spaces.

Let G be a region in the n-dimensional Euclidean space E^n with sufficiently smooth boundary, and $W_{p,a}^{(1)}$ the space of functions f with the norm

$$|u, W_{p,a}^{(1)}| = \sum_{k=0}^{1} {}' \sum_{k_1+\ldots+k_n=k} \left| r^a \frac{\partial^k x}{\partial x_1^{k_1} \ldots \partial x_n^{k_n}}, L_p(G) \right|$$

where r is the distance to the boundary.

We find conditions under which the functions of class $W_{p,a}^{(1)}$ have boundary values and investigate their properties, solve the inverse problem on the extension of functions and systems of functions from a manifold to the whole space, in particular, we study the question of extending functions and systems of functions in the best way from the point of view of the order of growth of the derivatives when approaching the manifold.

Ladyženskaja, O.A., Quasilinear equations of elliptic and parabolic types.

The report deals with linear and quasilinear equations of the second order with several independent variables of elliptic and parabolic types. The following questions are considered.

1) Generalized solutions of linear equations with unbounded coefficients and quasilinear equations. Necessary and sufficient conditions for the solutions to belong to the classes $C^{k,a}$ and C^k.

2) Conditions that functions, which satisfy certain integral inequalities, belong to $C^{0,a}$.

3) A priori estimates of the solutions of linear and quasilinear equations.

4) Global solvability of the first boundary value problem.

5) Smoothness and uniqueness in the small of generalized solutions of many-dimensional variational problems.

Lavrent'ev, M.A., Some boundary problems in potential theory.

1. Variational principles for potential flow of a fluid with axial symmetry.

2. Applications to the problem of jets.

3. New problems for the motion of a fluid with free boundaries.

4. Some properties of spatial jets. Construction of approximate solutions.

Lavrent'ev, M:M., On operator equations of the first kind and certain problems of potential theory.

In the investigation of a series of problems of mathematical physics it is natural to introduce a concept of correctness, which differs from the classical one. In this paper we formulate a new concept, correctness in the sense of Tihonov, and give some considerations on the natural investigation of the so called inverse problem of potential theory in the formulated statement. The solution of the problem discussed in the paper is equivalent to solving a certain operator equation of the first kind. Algorithms for solving linear and some non-linear operator equations of the first kind under the assumption of correctness in the sense of Tihonov are given in the paper. Then we intend to state theorems on the correctness in the sense of Tihonov of the following problems: the Cauchy problem for elliptic equations, the problem of defining an analytic function from its values on certain sets, some inverse problems of potential theory.

Leont'ev, A.F., On incomplete systems of Faber polynomials and some other functions.

We have proved the following theorem. Let $\{\lambda_n\}$ be a sequence of positive numbers with desity σ, D a region, which contains a vertical closed segment of length $2\pi\sigma$, and $f_n(z)$ a finite linear combination of functions in the system $\{e^{\lambda_n z}\}$. If the sequence $\{f_n(z)\}$ converges uniformly in D, then it converges in some half-plane $\mathrm{Re}(z) < a$. The following question arises: what can be said if in this theorem an arbitrary system $\{P_{\lambda_n}(z)\}$ is substituted for the system $\{e^{\lambda_n z}\}$? This problem has been solved if $P_n(z)$ (n = 0, 1, 2, ...) are classical polynomials (Čebyšev, Legendre and other polynomials). We now show how to solve it, when $P_n(z)$ are Faber polynomials or a few other analytic functions.

Linnik, Ju.V.,

Mal'cev, A.I., Theories of the first order of some classes of groups and rings.

Let K be a class of models of finite signature σ and a a property which the models in K can possess or not possess. By $T(K_a)$ we denote the set of closed formulas of the narrow calculus of predicates with equality, whose signatures belong to σ and which are true on each model of the class K that has the property a. Let r.e., p.r., fin denote the properties, recursively enumerable, primitively recursive and finite respectively.

For the case when the K is the class of all models with one binary predicate, P. Boot showed the following:

1) If $T(K) \subsetneqq W \subsetneqq T(K_{fin})$, then W is not recursive.

2) If $T(K_{r.e.}) \subsetneqq W \subsetneqq T(K_{p.r.})$ then W is not arithmetical.

4) There exists a non-empty, finitely axiomatized subclass of the class K that does not contain recursively enumerable models.

Using correspondencies between models and rings and groups it can be proved that the theorems 1-4 are true in the case when

  i) K is the class of all metabelian groups with the identity $x^p = 1$ (p odd prime)

  ii) K is the class of all rings with the identities $px=0$, $x^2 = x(yz) = 0$ and for some other K as well.

Manin, Ju.I., On the classification of formal abelian groups.

Formal abelian groups over an algebraically closed field of finite characteristic were classfied up to an isogeny by Dieudonné. The author proves that the set of classes of isoformal groups isogenous to a fixed group without unipotent component can be parametrized with the points of the quotient space of an algebraic manifold over the basic field by a finite group of isomorphisms. The dimension of this manifold is computed for simple groups.

Marcĕnko, V.A., The generalized spectral function.

Let the operator $1 = - d^2/dx^2 + g(x)$, $0 \leq x < \infty$, operate on all twice continuously differentiable functions with compact supports satisfying

$$u(0)h - u'(0) = 0.$$

Let $\omega(\lambda, x)$ be the solution of $1[\omega(\lambda, x)] = \lambda \cdot \omega(\lambda, x)$ satisfying $\omega(\lambda, 0) = 1$, $\omega'(\lambda, 0) = h$ and put

$$E_f(\lambda) = \int_0^\infty f(x) \omega(\mu, x)dx$$

for functions f in $L^2(0, \infty)$ with compact supports. Let Z be the linear hull in L^2 of the functions $E_f(\lambda) E_g(\lambda)$. Parseval's formula

$$\int_0^\infty f(x)g(x)dx = (R, E_f(\lambda) E_g(\lambda))$$

defines a linear functional R on Z, the generalized spectral function. If f has compact support and $E_f(\sqrt{\lambda}) \in L^1(0, \infty)$, then $E_f(\lambda)\omega(\lambda, x) \in Z$ and one has $f(x) = (R, E_f(\lambda)\omega(\lambda, x))$, which is the analogue of the ordinary eigenfunction expansion.

Mitropol'skiĭ, Ju.A., The method of integral manifolds in the theory of non-linear differential equations.

The method of integral manifolds presents a certain new approach in the qualitive theory of differential equations. As is well known, an individual solution of a system of differential equations is in general very sensitive to small variations of the right-hand sides of the equations.

In the method of integral manifolds we do not deal with individual solutions but with integral manifolds (not with curves but with hypersurfaces), which are more stable relative to small variations of the right-hand sides of the equations.

By means of this method non-linear differential equations containing a small parameter are successfully studied.

We establish effective criteria on the correspondence between integral manifolds of exact equations and of approximate ones.

<u>Naĭmark, M.A., On factor representations of a locally compact group.</u>

It is established that in the canonical decomposition into factor representations of a unitary representation of a locally compact group, almost all the factor representations involved in the decomposition are mutually disjoint. Hence we obtain for groups of type I: (1) A decomposition into mutually non-equivalent multiple irreducible representations; (2) An analogue of the Bochner theorem on integral representation of positive definite functions.

For groups not of type I the problem of studing the structure of factor representations arises, in particular to exhibit the conditions under which a continuous sum of factor representations is a factor representation. The author has obtained the following sufficient condition: If almost all factor representations involved in a continuous sum are mutually quasi-equivalent, then the continuous sum is also a factor representation. Necessary conditions have also been obtained.

<u>Nikol'skiĭ, S.M., On boundary properties of differentiable functions of several variables.</u>

Let $\mathscr{E}$ be a convex set of vectors $\vec{k} = (k_1, \ldots, k_n)$ with integral non-negative coordinates such that $\mathscr{E}$ contains with $\vec{k}$ its projections on the coordinate hyperplanes of dimension $\leq n$. Let $\vec{k}^1, \ldots, \vec{k}^N$ be a minimal system of vectors of $\mathscr{E}$ such that an arbitrary vector can be represented as

$$\vec{k} = \sum_1^N \lambda_s \vec{k^s}, \quad \lambda_s \geq 0, \quad \sum_1^N \lambda_s \leq 1 .$$

For a function φ and its partial derivatives of order $\vec{r} \in \mathscr{E}$ satisfying

$$\infty > E_g(\varphi) = \int_g \sum_{\vec{k}, \vec{l} \in \mathscr{E}} L_{\vec{k} \vec{l}} \varphi^{(\vec{k})} \varphi^{(\vec{l})} dg > \sum_{s=1}^N \int (\varphi^{(\vec{k^s})})^2 dg$$

$$L_{\vec{k} \vec{l}}(x) = L_{\vec{l} \vec{k}}(x) \qquad |L_{\vec{k} \vec{l}}(x)| < M \text{ for } g \subset R_n$$

we define in terms of function theory a complete system of stable boundary

values on the boundary Γ of the region g. We introduce the concept of regular boundary point. The projections on the coordinate hyperplanes of dimension n-1 of the set of irregular points of Γ must have (n-1)-dimensional measure zero. Applications are given to direct variational methods for solution of boundary problems for hypoelliptic partial differential equations.

Novikov, S. P. , Smooth manifolds with common homotopy type.

A study is performed on smooth simply connected manifolds M^n having the same homotopy type and stable normal bundle. A correspondence is established between these manifolds and the homotopy group $\pi_{N+n}(T_N)$ where T_N is the Thom space of the normal bundle of the manifold. Some qualitative results are obtained (the number of such manifolds is finite) and some examples are examined. The results are applied to the following problems:

1) What bundle SO(N) on M^n is normal for some M_1^n homotopically equivalent to M^n and smooth (M^n may be combinatorical).

2) In what cases is the sum $M^n \# S^n$ diffeomorphically equivalent to the manifold M^n, where $\widetilde{S}^n$ is Milnor's non-standard sphere.

The correspondence between the homotopic and the diffeomorphic problems is established on one hand by means of algebraic methods, on the other by the t-regularity, Moore's construction theory and the results of Smale-Wallis. In the treatment of the concrete examples multiplication in stable homotopy groups is used.

Oleĭnik, O. A. , On the equations of a boundary layer.

In the lecture are considered boundary problems for the equations of a boundary layer and some quasi-linear parabolic equations degenerating for certain values of the unknown function.

Conditions for the existence of generalized and classical solutions of boundary problems are studied as well as qualitative properties of such solutions.

Ovsjannikov, L. V. Application of the theory of Lie groups to some partial differential equations.

The theory of group properties of differential equations involves the following problems:

1) Investigation of the largest Lie group admitted by a given system S.

2) Group-theoretical classification of a system S of given type.

3) Application of a given group, admitted by a system, to the construction of solutions.

4) Investigation of systems which admit a given group or, more generally, a group with given properties.

The author has obtained a group-theoretical classification of the equations of non-linear heat flow, the systems for the adiabatic flow of gases, Čaplygin's equation and others. He has analyzed several special classes of solutions: invariant, partially invariant and imprimitive, and also investigated the relations between them. He has also constructed a geometrical invariant form in the Lie algebra of the group, permitted by a general second order differential operator.

Pasynkov, B.N., see p. 3.23.

Pogorelov, A.V., On the isometric immersion of a two-dimensional Riemannian manifold into a three-dimensional Riemannian space.

Let R be a complete three-dimensional Riemannian space and M a closed Riemannian manifold homeomorphic to a sphere, with Gaussian curvature everywhere greater than a constant c. Then if the curvature of the space R is everywhere less than c, M admits an isometric immersion into R as a regular surface F. Moreover, this immersion can be so realized that a given two-dimensional element a of the manifold M (a point S and the pencil of directions at S) coincides with a given two-dimensional element a' of R, isometric to a, and that the surface F is placed along a given side of the surface element a'. The surface F whose existence is established is uniquely determined by the two-dimensional element a'.

Ponomarev, V.N., Projective spectra and topological spaces.

Consider the projective spectra

$$(1) \qquad S = \{a, \overline{\omega}_a^{a'}\}, \quad \widetilde{S} = \varprojlim S$$

from the (complete) simplicial complexes a with the simplicial projections $\overline{\omega}_a^{a'}$.

One defines in a natural way the weakening of a spectrum (1), in particular the complete weakening, i.e. the spectrum

$$(2) \qquad S^\cdot = \{\dot{a}, \omega_a^{a'}\}$$

where $\dot{a}$ is the zero-dimensional complex, which consists of all vertices of the complex a; the projections in (2) and in (1) are the same.

Each vertex $e_a \in a$ defines the closed set $\emptyset_{e_a}$ of all $\xi \in \widetilde{S}$ with $e_a \leq t_a \in \xi$.

Given a, these $\emptyset_{e_a}$ form a covering φ_a of the space $\widetilde{S}$. The spectrum S is called uniform, if in every open covering of the space $\widetilde{S}$ some φ_a is inscribed.

Theorem 1. Among all regular spaces all paracompact spaces and only those are

a) limits of uniform spectra

b) images of (perfectly) zero-dimensional spaces by irreducible perfect mappings (a space is perfectly zero-dimensional if in each of its open coverings it is possible to inscribe a disjoint covering).

If X is paracompact, the set of all decompositions (locally finite coverings, consisting of the closures of disjoint open sets) defines the "maximal" spectrum S; the limit space $\tilde{S}^\bullet$ of the complete weakening $S^\bullet$ is called the absolute space of the space X; this is a perfectly zero-dimensional space $X^\bullet$.

There exists a natural irreducible perfect mapping $\pi_x: X^\bullet \to X$ (the transition from the spectrum S to the spectrum $S^\bullet$).

If X_1 and X_2 are paracompact, a perfect (multiple-valued) mapping f: $X_1 \to X_2$ exists, when and only when X_1 and X_2 have the same absolute space: $X_1^\bullet = X_2^\bullet$; all these mappings are given by the formula $f = \pi_{x_2} f^\bullet \pi_{x_1}^{-1}$, where $f^\bullet$ is an arbitrary homeomorphism of the absolute space onto itself. The mapping f is single-valued if the spectrum S_1 is a weakening of the spectrum S_2; then f is generated by the natural transition from the spectrum S_1 to the spectrum S_2.

Pontrjagin, L.S., A statistical problem in the theory of optimal control.

We consider a stochastic object when the information on the position of the object in the phase space R is given by the Fokker-Planck-Kolmogorov equations. In the same space R we consider a deterministic motion of a k-dimensional plane P of the space R for which the plane remains parallel to itself all the time. The probability of contact in an interval of time between the stochastic object and an ε-neighborhood of the moving plane is calculated. The principal term of this probability is found under the condition that ε is small. The work has been carried out in collaboration with E.F. Miščenko.

Postnikov, M.M., Axiomatic definitions of cohomology groups.

Systems of axioms describing the cohomology groups of arbitrary spaces are formulated and discussed.

Rosanov, Ju.A., see p. 3.23.

Sarymsakov, T.A., Topological semi-fields and some of their applications.

In my publications together with M.Y. Antonovskiǐ and V.G. Boltjanskiǐ the concept of topological semi-fields is introduced, their properties and structure are studied and their applications to some questions of functional analysis and topology are also presented. In particular the following metrization theorem is proved: "In order that a topological space be metrizable over any semi-field, it is necessary and sufficient that the space be completely regular". We also prove a theorem generalizing a well-known theorem of A.N. Kolmogorov

giving necessary and sufficient conditions for the normability of a linear space. The following questions are discussed: the convergence in spaces metrized by a semi-field, completeness, compactness, the well-known theorems by Cantor and Hausdorff, and finally Banach's theorem on the open mappings or ordinary normal spaces is carried over to the case of normal spaces over semi-fields. Further, the use of topological semi-fields permits the generalization of the operator theory abstract ergodic theorems, due to Eberlein, and also a generalization of the theorem of the ergodic principle of Markov processes, originated by Markov himself.

Sevast'janov, B.A., Limit theorems for branching stochastic processes with diffusion,

The author considers a branching stochastic process for particles moving in a bounded region G with an absorbing boundary. Let $\mu_t(A)$ be a stochastic variable equal to the number of particles in the set $A \subset G$ at the moment t. These stochastic variables determine a stochastic measure $\mu_t(\cdot)$. For the characteristic functional $H_t(\varphi(\cdot)) = E(\exp i \int_G \varphi(x)d\mu_t)$ of this stochastic measure the author derives a non-linear parabolic partial differential equation. Define $\xi_t(\cdot) = \mu_t(\cdot)/E\{\mu_t(G) > 0\}$. Then the conditional distribution of $\xi_t(\cdot)$ under the condition $\xi_t(G) > 0$ will converge weakly towards a limit distribution when $t \to \infty$. The character of the limit distribution depends on the behaviour of $E(\mu_t(G))$ when $t \to \infty$. If $\lim_{t\to\infty} E(\mu_t(G)) \neq 0$ and $\neq \infty$, then the characteristic functional of the limit distribution equals

$$H(\varphi(\cdot)) = (1 - i(\int_G \varphi(x) \, a(x)dx) (\int a(x)dx)^{-1})^{-1},$$

where $a(x)$ is the first eigenfunction of a certain boundary problem.

Sirazdinov, S.H., On strong convergence of the distributions of sums of independent terms.

We consider a sequence of equally distributed independant stochastic variables with mean value zero and with dispersion equal to 1. We study the convergence of the distribution function of the sum normed in the usual way to a normal distribution function.

This type of convergence is equivalent to convergence of the corresponding probability functions to a normally distributed probability function of stochastic variables. Here the convergence is uniform on all Borel measurable sets of the real line.

When a third moment of the initial stochastic variables exists we exhibit an

exact expression for the principal remainder term. We also consider some other questions connected with this problem.

Širšov, A. I. , Quelques questions algorithmiques de la théorie des anneaux.

Les notions du groupe libre et du produit libre des groupes et aussi les résultats liés avec ces notions ont des analogues dans la théorie des algèbres. On sait, par exemple, que chaque sous-algèbre de l'algèbre libre de Lie elle même est libre. Ce résultat est analogue au théorème connu de Nilsen-Schreier de la théorie des groupes. Les résultats de A. T. Gaïnov de sous-algèbres de la somme libre commutative ou anticommutative des algèbres sont analogues au théorème de A. G. Kuroš de sous-groupes du produit libre des groupes.

Sous influence de cette analogie existait l'hypothèse qu'on décrit les sous-algèbres de la somme libre de Lie des algèbres de Lie de la même manière.

Dans cette nouvelle on construit un exemple qui montre que ce n'est pas ça.

Les méthodes employées dans ce travail permettent de résoudre une série de problèmes algorithmiques et en particulier le problème de l'identité pour les algèbres de Lie avec une corrélation.

Skljarenko, E. G. , On topological construction of locally bicompact groups and their quotient spaces.

The decomposition of a bicompact group into a Lie series, proposed by L. S. Pontrjagin, is extended to quotient spaces of locally bicompact groups. By means of such a decomposition the following questions are investigated: construction of quotient spaces of dimension zero; properties of the fibres; factorization of a quotient space as a direct product; the countable character of connected finite-dimensional bicompact quotient spaces. A few applications to dimension theory are given.

Skornjakov, L. A. , Locally bicompact biregular rings.

Theorem. A locally bicompact biregular ring R decomposes into a topological direct sum $R = A \dotplus D$, where A is the direct sum of a finite number of simple rings with unit element, and D is a local direct sum of discrete rings relative to finite biregular subrings.

From this it is easy to derive the well-known result by Kaplansky on the construction of locally bicompact strongly regular rings (Kaplansky, I. , Am. J. Math. , 73 (1951), 20 - 24).

The proof is based on a study of the central idempotents of the ring R. We note the following auxiliary proposition:

Theorem. Let R be a topological ring, S an open bicompact subring, and

$\{\varepsilon_i\}$ a bounded system of mutually orthogonal central idempotents of the ring R.

Then the closure of the subring $\sum Re_i$ coincides with the local direct product of the rings Re_i relative to Se_i.

Moreover, in the proof we use some properties of the biregular radical due to V.A. Andrunakievič (Mat. sb., 39 (1956), 447 - 464).

Sobolev, S.L., Les formules de l'intégration numérique sur la surface d'une sphère.

La formule

$$\int_0^\pi \int_0^{2\pi} f(\vartheta, \varphi) \sin \vartheta\, d\vartheta\, d\varphi = \sum_k C_k f(\vartheta_k, \varphi_k) \qquad (1)$$

sera exacte pour les différentes fonctions sphériques si l'on suppose que cette formule sera invariante pour les rotations de la sphère d'un groupe G.

Au moyen de lemme sur l'indépendance linéaire des fonctions:

$$(x_k + iy_k)^n, \quad k = 1, 2, \ldots, \quad 2n + 1, \qquad (2)$$

où (x_k, y_k, z_k) est un système orthogonal et les z_k sont différentes, on établit le nombre de telles harmoniques sphériques pour lesquelles pas toutes les formules (1) invariantes relativement aux groupes des polygones réguliers sont vraies.

On construit les formules du type (1) valables pour le plus grand nombre des harmoniques sphériques.

Sobolev, S.L., Quelques questions de la théorie des intégrations numériques et de l'interpolation pour les fonctions des plusieurs variables indépendantes.

On étudie le problème du minimum pour la fonctionelle des fautes:

$$(1, f) \equiv \int f\, d\Omega - \sum C_k f(x^{(k)})$$

ou $(j, f) \equiv f(z) - \sum C_k f(x^{(k)})$

dans l'espace $W_2^{(m)}$ des fonction avec les dérivées d'ordre m à carré sommable, où $m > \frac{n}{2}$, et n est le nombre de dimension.

On construit sur la sphère unitaire dans $W_2^{(m)}$ les fonctions f pour lesquelles la fonctionelle atteint son maximum. Cela donne une évaluation exacte des fautes et montre les voies de la construction des formules les plus économiques ou les plus voisines aux plus économes.

Suprunenko, D. A., On periodic matrix groups.

Theorem 1. Let P be an arbitrary complete field, and Γ a group of matrices over P, possessing a completely reducible normal divisor A. If the factor group Γ/A is periodic and has no elements whose orders divide the characteristic of the field P, then the group is completely reducible.

In particular, if P is an arbitrary field with zero characteristic, then the group of matrices over P possessing a completely reducible normal divisor whose corresponding factor group is periodic is completely reducible.

Theorem 2. Let $\triangle$ be an algebraically closed field of zero characteristic, and Γ a maximal solvable subgroup of GL(n, $\triangle$). Then the maximal periodic subgroups of the group Γ are conjugate in Γ.

Suvorov, G.D., Topological mappings of plane regions with variable boundaries.

The author lays the foundations of the theory of topological mappings $T =$ $= f_1(x, y) + if_2(x, y)$ of a family of plane regions onto another such family, under the assumption that the mappings have generalized partial derivatives of the first order with respect to x and y (in the sense of S. L. Sobolev) and bounded Dirichlet integrals.

In the metric part of the theory it is essential to find estimates of the distortion of distances for mappings of regions and closed (by prime ends) regions. These estimates are used to prove results on equicontinuity and equi-openness. Combining these results with the topological theory of prime ends for a sequence of plane regions, converging to a nucleus (cf. our paper in Mat. Sb., 33(75):I (1953), 73-100 or in American Mathematical Society Translations, S. 2, Vol. 1 (1955), 67-93), we prove the fundamental theorem on the correspondence of boundaries for topological mappings (of the class referred to) of regions with variable boundaries.

The author carries out different applications of the theory to questions of convergence of a sequence of mappings.

Taĭmanov, A.D., The characteristic of elementary classes of models.

A set of propositions F of the language L is called a generalized atomic set, if:

1) $f \in F$ and $g \cong f$ implies $g \in F$

2) the identically false formula belongs to F

3) F is closed under the operations $\wedge$, $\vee$ and substitution of variables

4) F contains atomic formulas.

Every set F of generalized atomic formulas defines in L an F-topology (L is

the class of all models with given signature). The class K, $K \subset K_1 \subset L$ is F-elementary relative to K_1 if there exists an elementary class $L_1 = \Sigma^*$, $\Sigma \subset F$ and $L_1 \cap K_1 = K$.

Theorem: The class K, $K \subset K_1$ is F-elementary relative to K_1 if and only if K is F-closed in K relative to the F-topology.

If $K_1 = L$, we obtain the characteristic of the elementary classes and of the models describable by means of the system of propositions with given quantifiers; if also the signature is finite, then we obtain theorems from (1). In the same terms criteria of completeness are obtained, leading to the completeness of some theorems.

Talaljan, A. A., Representations of measurable functions by trigonometrical series.

By means of some lemmas about properties of trigonometrical polynomials the following theorems are proved, which generalize the wellknown theorems of Menšov.

Theorem 1. There exists a trigonometrical series

$$(1) \quad \sum_{n=0}^{\infty} (a_n \cos n x + b_n \sin n x)$$

which has a subseries

$$(2) \quad \sum_{k=0}^{\infty} (a_{n_k} \cos n_k x + b_{n_k} \sin n_k x)$$

converging to any given a. e. finite measurable function.

Theorem 2. For each measurable function f(x), either a. e. finite or equal to $+\infty$ or $-\infty$ on a set of positive measure, there exists a trigonometrical series (1) which converges to f(x) a. e. in the set where f(x) is finite and converges to f(x) in measure on the set where f(x) assumes the values $+\infty$ or $-\infty$.

Timan, A. F., On some new questions in the theory of approximation of functions of a real variable.

Some inequalities in direct theorems of constructive function theory in the case of approximation in the whole real line. Questions connected with the approximation of functions by algebraic polynomials on a finite interval. Inverse theorems, their development and refinement. Best approximation on compact sets, ε-entropy and width of sets. Mutual deviation of some classes of continuous functions given in a separable metric space. Some new results concerning linear processes for approximating functions by polynomials.

(for p = 1 : $S^*_\Gamma \partial g(\xi) = 0$ for $\mu_\Gamma(\xi) > a$)

in any subcone Γ' of $\mu_\Gamma(\xi) > 0$.

Conversely: if $g(\xi)$ satisfies these conditions then $f(z)$ is holomorphic in $R^n + i\Gamma$ and the estimate (1) is true (for p = 1 with $\epsilon = 0$). Analogous results are true for non-convex and even for non-connected cones Γ.

Volkovyskiǐ, L.I., Questions of approximation and algebraic theory for some classes of functions on Riemann surfaces.

By the young participants of the seminar on the theory of functions at the Perm State University, the following was established.

1. The theorems on uniform approximation of functions which are continuous on some set M and analytic at inner points of M by rational functions are still valid for arbitrary closed Riemann surfaces, and also if the approximating and approximated functions are supposed to be generalized analytic or quasi-conformal. (S. Ja. Gusman)

2. The theorems of Riemann-Roch and Abel can be transferred to generalized analytic functions. Hereby the algebraic theory is intimately connected with the theory of boundary problems for these functions. (Io. L. Rodin)

3. In the Teichmüller space of closed Riemann surfaces the set of points corresponding to the quasiconformal mappings of a given closed Riemann surface with constant characteristic $p(z)$ and arbitrary distribution of the characteristic $\theta(z)$ fills a closed sphere of the dimension of the space. (V.G. Šeretov). The result is apparently preserved also for a given variable distribution $p(z)$.

Vrkoč, I., Stability under constantly acting perturbations.

Let a system of differential equations $\dot{x} = f(t, x)$, $f(t, 0) = 0$ be given with $f(t, x)$ continuous. The solution $x \equiv 0$ is stable under constantly acting perturbations if and only if there exists a Ljapunov function $V(t, x)$ defined in the region $t \geq 0$, $0 \leq \|x\| \leq a$ (a > 0) where the following conditions are fulfilled:

(1) $V(t, x)$ is positive definite

(2) $V(t, x)$ has an infinitesimal upper limit

(3) There exists a function $U(\eta)$, $U(0) = 0$, $U(\eta) > 0$ for $\eta > 0$ such that

$$W(t, x) = \frac{dV}{dt} + U(\|x\|)\sqrt{\sum \left(\frac{\partial V}{\partial x_i}\right)^2} \leq 0.$$

It is also possible to introduce the concept of strong asymptotic stability under constantly acting perturbation for which the same theorem is valid with the only change that we suppose $W(t, x)$ to be negative definite in the condition (3).

Gnedenko, B.V., On some extremal problems in queuing theory.

1. On an extremal property of the Poisson flow.

2. On the optimal distribution of service systems of various capacities for a problem with losses.

3. A problem due to G.P. Klimov: determination of the extremal input flow of a semi-manufactured article to obtain a maximal output of finished products; to obtain a minimal number of losses.

Linnik, Ju.V., On similar regions in mathematical statistics.

1. Similar regions of linear form for iterated sampling.

2. Construction of a form of distribution, generalizing the Neumann structures for one parameter.

3. On the case of several parameters.

Pasynkov, B.N., On universal spaces.

The following theorem is true. For arbitrary $n = 0, 1, 2, 3, \ldots$ and arbitrary cardinal number $\tau \geq c$ there exists a bicompact set $P^{n\tau}$ of weight τ and dim $P^{n\tau} = n$ which is universal for a certain class of completely regular spaces, n-dimensional in the sense of ind. This class contains in particular:

(a) all bicompact sets X of weight τ and dimension dim $X \leq n$ which can be zero-dimensionally mapped on some compactum;

(b) all metric spaces R of weight τ and dimension dim $X \leq n$.

The properties of $P^{n\tau}$:

(1) $P^{n\tau}$ can be zero-dimensionally mapped on a compactum;

(2) dim $P^{n\tau}$ = ind $P^{n\tau}$ = Ind $P^{n\tau}$;

(3) $P^{n\tau}$ is a homogeneous space;

(4) $P^{n\tau}$ is linearly connected;

(5) $P^{n\tau}$ is an (n+1)-fold image of a zero-dimensional bicompact set;

(6) $P^{n\tau}$ is a continuous image of $\mathcal{D}^\tau$;

(7) $P^{n\tau}$ is the limit of an inverse spectrum of n-dimensional polyhedra with simplicial projections.

Rosanov, Ju.A., On the question of equivalence of probabilistic measures corresponding to Gaussian stationary processes.

We consider a stationary Gaussian process $\xi_0(t)$ and the corresponding probabilistic measure P_0 in the space of all real functions of t, $0 \leq t \leq T$, defined on a σ-ring generated by cylindrical sets. We characterize the class of stationary Gaussian processes $\xi(t)$ such that the corresponding measure P is absolutely continuous with respect to P_0.